MIND PERFORMANCE HACKS™

Tips & Tools for Overclocking Your Brain

O'REILLY®

Ron Hale-Evans

HACK #35

Put Down That Calculator

You don't need a calculator to do simple math! Learn a few tricks, and with a little practice, you'll be surprised how much arithmetic you can do in your head.

Most people need a calculator to do even simple arithmetic. There's nothing wrong with that, but if a calculator isn't available, it can become a problem. And even if you don't count the time to find the calculator, mental arithmetic can actually be faster than a calculator, too.

In Action

Entire books have been written on mental arithmetic, so we're not going to cover everything in this hack. This hack covers some typical techniques useful in their own right, and some of the other hacks in this chapter are also useful in doing mental mathematics. If you find this hack interesting and useful, you can check out one of the many books on the subject, some of which are listed at the end of this hack.

You should start at a level that's not frustrating for you. If you reach for a calculator to multiply 8 × 7, start by learning the multiplication tables. Use paper and pencil at first, and *check your work*.

Rearrange. Suppose you need to add the following numbers:

```
9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
```

You *could* add 9 + 8 to get 17, and then add 7 to that to get 24, and so on. But it's much easier to rearrange the addition:

```
9 + 1 + 8 + 2 + 7 + 3 + 6 + 4 + 5
```

Each of the first pairs of numbers adds up to 10. So, we have the following easy addition:

```
10 + 10 + 10 + 10 + 5
```

which is 45.

In addition to rearranging to find 10s (or 20s), rearranging numbers so that they're in descending order tends to help. For instance, suppose you're adding the following numbers:

```
  1100000
   270000
  3300000
+   30000
---------
```

It's probably easier to rearrange that as follows:

```
 3300000
 1100000
  270000
+  30000
```

It's easier because you don't need to keep track of as many nonzero digits while adding 3,300,000 and 1,100,000. Adding 270,000 and 30,000 will also help, so you're left with 4,400,000 + 300,000—an easy sum that totals to 4,700,000.[1]

Put down that burden. When you learned to do paper-and-pencil multiplication in school, you probably learned to work from right to left, carrying as you went:

```
 12
  841
 x 74
 3364
5887
62234
```

Notice that you had to carry twice, once in multiplying the 4 in 74 by the 4 in 841, and once in multiplying the 7 by that 4. If you try to do this mentally, you'll have to keep track of multiple numbers between steps. For instance, when multiplying the 7 by the 8, you need to remember the 3,364 from the first multiplication, the 87 you've already figured out, and the carried 2.

Instead of working right to left, let's work left to right, just multiplying one of the six pairs of digits at a time. We're not doing this just to be different; rather, we want to limit the amount of information we need to keep track of.

Before we attempt our previous example, let's do a simpler problem. Multiplying a two-digit number by another two-digit number turns out to be particularly nice. Suppose we need to multiply 42 × 29. We multiply each pair of numbers (keeping track of the powers of 10), starting at the left, and keep a running total:

The individual calculations look like this:

`40 x 20 = 800`
`40 x 9 = 360`, and `800 + 360 = 1,160`
`2 x 20 = 40`, and `1,160 + 40 = 1,200`
`2 x 9 = 18`, and `1,200 + 18 = 1,218`

Notice that we have to remember only one number between steps; this remains true even for larger problems. Of course, there's nothing wrong with writing down that number if paper and pencil are handy, and you'll probably find this method is still easier and less error prone that the usual method.

> Because you need the number for only a short time, the mnemonics from Chapter 1 probably aren't necessary here.

Moving from higher numbers to lower ones tends to work better because it's easier to add a small number to a large one. As a bonus, if you need only an estimate, you can stop after doing the first few multiplications.

Now, back to our initial example. It contains pairs of digits:

The calculations look like this:

```
800 x 70 = 56,000
800 x 4 = 3,200, and 56,000 + 3,200 = 59,200
40 x 70 = 2,800, and 59,200 + 2,800 = 62,000
40 x 4 = 160, and 62,000 + 160 = 62,160
1 x 70 = 70, and 62,160 + 70 = 62,230
1 x 4 = 4, and 62,230 + 4 = 62,234
```

Of course, this is the same answer that we got before; doing a problem two ways is a good way to check it.[2]

> "Calculate Mental Checksums" **[Hack #38]** and "Estimate Orders of Magnitude" **[Hack #41]** discuss other ways of checking your work.

Look for friendly numbers. Which addition problem would you rather do: 79 + 87 or 80 + 86? Probably the second; it's easier because the 80, ending in 0, is a friendly number **[Hack #36]**. For addition, numbers ending in 0 are friendly because adding the corresponding place is trivial (adding 0 to a number doesn't change it). Thus, we can mentally add the tens place (8 + 8 = 16) and then just append the 6 for the ones place to get the answer, 166.[3]

The trick for more difficult addition problems is to change the problem without changing the answer so that we have friendly numbers. For instance, if we had `79 + 87`, we'd notice that 79 is near the friendly number 80. To turn 79 into 80, we have to add 1, so to keep the answer the same, we need to subtract 1 somewhere. Let's subtract 1 from the other number, 87, to get 86 and do `80 + 86 = 166`, as in the previous example.

> Alternatively, you can add `80 + 87 = 167`, and subtract the 1 from the result.

The same principle works with multiplication.[4] Suppose I ask you to compute `300 × 70` in your head. Multiplying `3 × 7 = 21` and then adding the three 0s, you easily get 21,000. Again, the 0s at the end make the multiplication easy.

If we need to multiply `302×69`, we can think as follows:

```
302 x 69 = (300 + 2) x (70 - 1)
```

Now we can do the same cross-multiplication we did before, but with bigger chunks:

`300 x 70 = 21,000`
`300 x -1 = -300` and `21,000 - 300 = 20,700`
`2 x 70 = 140` and `20,700 + 140 = 20,840`
`2 x -1 = -2` and `20,840 - 2 = 20,838`

Numbers that end in 0 are the friendliest, but factors of 100 **[Hack #36]** are pretty friendly, too. For instance, if you think about the factors of 100, you'll find that 25 is a friendly number, and 75 is at least the friend of a friend. Then, the example we used earlier, `841 × 74`, looks like this:

```
841 x 74 = 841 x (75 - 1) = 841 x 75 - 841
```

Remember the minus 841, and let's do `841 x 75`:

```
841 x 75 = 841 x 3 x 25
         = 2523 x 25
         = (2524 - 1) x 25
         = 2524 x 25 - 25
         = 2524 x 100 / 4 - 25
         = 252400 / 4 - 25
         = 63100 - 25
         = 63075
```

Finally, subtract the leftover 841, part by part:

```
63,075 - 800 = 62,275
62,275 - 40 = 62,235
```

```
62,235 - 1 = 62,234
```

which is a third confirmation of the result!

How It Works

All of these tricks rely on basic properties of integers. For instance, the first multiplication example we gave was 841 × 74. We figured that out by using:

```
841 x 74 = (800 + 40 + 1) x (70 + 4)
         = 800 x (70 + 4) + 40 x (70 + 4) + 1 x (70 + 4)
         = 800 x 70 + 800 x 4 + 40 x 70 + 40 x 4 + 1 x 70 + 1 x 4
```

The first line uses the definition of the decimal number system [Hack #40], and the remaining lines are repeated applications of the distributive property.

In Real Life

Suppose you're playing a card game [Hack #67] with a 52-card deck. Your final score is based on your final cards, and each number card is worth its number (so the 4 of hearts is worth 4 points), with an ace worth 1 point and each face card worth 10 points. Your hand is shown in Table 4-1.

Table 4-1. The cards you're dealt

Suit	Cards
Spades	2, 7
Hearts	A, 8
Diamonds	2, 3, 8, 10
Clubs	A, 2, Q

The rearranging method works especially well because we have actual cards to rearrange.

Put the 10 and queen aside: they're worth 20 points. Then, arrange cards in the following groups (Figure 4-1):

- Ace of hearts, 2 of spades, 7 of spades = 10 points
- 8 of hearts, 2 of diamonds = 10 points
- 8 of diamonds, 2 of clubs = 10 points

There are 50 points so far; we just need to add the remaining cards, which are the ace of clubs and the 3 of diamonds, for 4 more points. Your total score is 54 points, and you've probably figured this out so much faster than anyone else that you can double-check the rules to make sure the three 2s don't get you some kind of bonus.

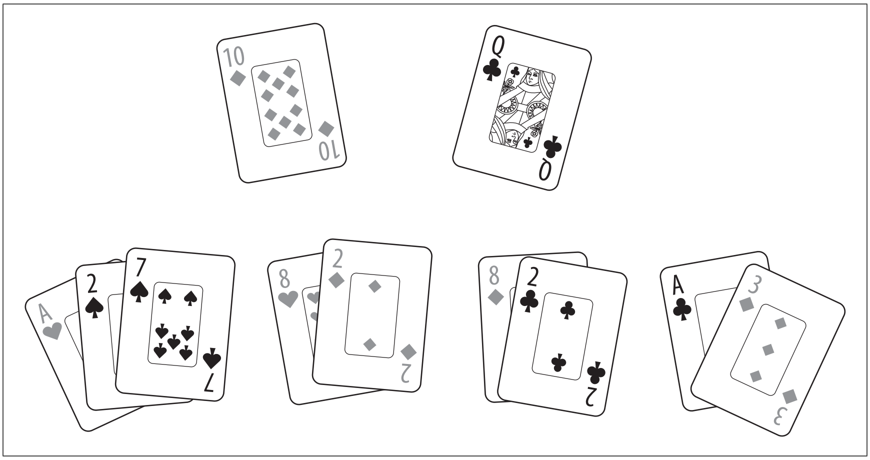

Figure 4-1. Grouping the cards in your hand

End Notes

1. Sticker, Henry. 1955. *How to Calculate Rapidly*. Dover.
2. Julius, Edward H. 1996. *More Rapid Math: Tricks and Tips*. Wiley.
3. Kelly, Gerard W. 1984. *Short-Cut Math*, Chapter 2. Dover.
4. Julius, Edward H. 1992. *Rapid Math Tricks and Tips*. Wiley.

See Also

- Gardner, Martin. 1989. *Mathematical Carnival*. Mathematical Association of America. Chapters 6 and 7 discuss calculating prodigies and some of the tricks they used.
- Doerfler, Ronald W. 1993. *Dead Reckoning: Calculating Without Instruments*. Gulf Publishing Company. This book is a bit more advanced. In addition to the basic operations, it covers extracting roots, and even higher mathematical functions, such as logarithms and trigonometric functions.

—Mark Purtill